

Market Interpretation and Uncertainty in Monetary Transmission[†]

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Preliminary, comments welcome!

Abstract

Monetary policy decisions affect economic activity largely by shaping private-sector expectations. The difficulty of anticipating how agents will interpret a decision is thus an important source of uncertainty about how strongly policy affects the economy (transmission uncertainty). I study a model in which agents infer the central bank's persistent real-time economic assessments from its decisions in a way that the central bank cannot perfectly predict. Systematic policy shapes this inference, making the extent of transmission uncertainty endogenous. I show that, under discretion—when the central bank chooses its preferred rate after observing its assessments—transmission uncertainty attenuates the policy response, in line with the familiar cautionary principle. However, this caution is inefficient. First, the discretionary rule is too cautious in its overall scale: a stronger policy response improves stabilization without increasing exposure to interpretation risk. Second, a self-reinforcing feedback loop between uncertainty and caution can generate arbitrarily large, avoidable losses.

Keywords: Monetary policy, transmission uncertainty, expectations, commitment, policy caution

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Uncertainty—about the state of the economy, the economy’s structure, and the inferences that the public will draw from policy actions or economic developments—is a pervasive feature of monetary policy.

— Chairman Ben Bernanke (2007)

1 Introduction

Most major channels of monetary transmission operate through expectations of future economic conditions, particularly the future path of policy. For one, mortgage rates, corporate borrowing costs, and asset values are all forward looking and thus shaped by financial market expectations of policy. These, in turn, influence both households’ decisions (e.g., consumption and housing) and firms’ investment choices. Furthermore, their decisions are directly shaped by their own expectations of future income, demand, and financing conditions. Anticipating the effects of a policy decision thus requires anticipating how it will change the expectations underlying both financial prices and spending decisions.

However, the response of these expectations to policy is quite uncertain. Market prices and surveys provide information about prevailing expectations, but do not directly reveal how those expectations would change following a particular policy decision. Thus, the difficulty of anticipating these revisions constitutes a form of monetary transmission uncertainty: uncertainty about how strongly the central bank’s own intervention will affect the economy. This source of uncertainty has a crucial feature that distinguishes it from more standard sources (e.g., uncertainty about the elasticity of intertemporal substitution in a standard New Keynesian model): it is itself shaped by systematic policy.

This paper studies the implications of this interpretation-driven transmission uncertainty by extending the framework developed in [Jimenez-Gimpel and Caravello \(2026\)](#). The model features a central bank whose decisions are based on private and imperfect real-time estimates of demand and natural output. Markets observe the fundamentals themselves but must infer the central bank’s estimates from its policy decisions. Since these estimates are persistent, learning about them helps markets predict future policy and output. A rate decision reflects both estimates, and the rule’s relative responses determine what markets infer and hence how a policy surprise affects contemporaneous output. The central bank is uncertain about the market’s relative uncertainty over those estimates. This relative uncertainty, which I call the *interpretation state*, affects the market’s update following a rate surprise. By changing the relative weight of the two estimates, the policy rule changes not only market inference itself but also how it depends on the interpretation state. Thus, systematic policy shapes how uncertainty about the interpretation state translates into monetary transmission uncertainty.

Main results. The analysis first characterizes the discretionary response to uncertainty and then assesses the efficiency of the resulting allocation. In a discretionary equilibrium, the central bank chooses its preferred rate after observing its assessments, taking the market’s inference rule—and thus the extent of transmission uncertainty—as given. Uncertainty about market interpretation then leads the central bank to respond more cautiously to its assessment of natural output ([Proposition 1](#)). Two forces account for this caution. Uncertainty about interpretation raises average transmission at a given policy rule. In addition, states in which transmission is stronger receive more weight in the central bank’s first-order condition. Both forces attenuate the desired response to its natural-output assessment. Thus, if policy is discretionary, transmission uncertainty induces caution, in line with [Brainard \(1967\)](#). However, the policy response and the uncertainty that motivates it are jointly determined.

This joint determination can make caution self-reinforcing. Concretely, a weaker reaction to perceived natural output fluctuations can make the market’s response to a rate surprise more sensitive to the unknown state, increasing the dispersion of effective transmission and thus validating the caution that caused the weaker response. Similarly, a stronger natural-output response can decrease that dispersion and thus validate itself. [Example 1](#) illustrates how this feedback can support multiple discretionary equilibria with different responses to natural output assessments.

In sharp contrast with [Brainard \(1967\)](#), discretionary caution is inefficient, and it is so along two different margins. First, discretionary policy is too cautious in its overall scale. This is because it does not account for the fact that markets adjust their inference to offset the proportional change in the rule, so a more aggressive overall reaction improves stabilization without increasing the central bank’s exposure to interpretation risk ([Proposition 2](#)). Second, the feedback between caution and uncertainty can sustain an inefficient composition of the policy response, with too little weight on natural-output assessments relative to demand assessments. In some interpretation states, markets then read small rate surprises as evidence of large changes in the central bank’s natural-output assessment. Policy reactions to demand-assessment errors can consequently generate large belief revisions and output fluctuations. This amplification can produce arbitrarily large losses even though the variance of the interpretation state remains bounded, while a fixed positive rule guarantees bounded losses ([Proposition 3](#)).

Related literature. This paper builds on the framework of [Jimenez-Gimpel and Caravello \(2026\)](#), in which policy decisions convey information about the central bank’s private and persistent real-time assessments of the economy, creating a belief-revision channel of monetary transmission. That paper identifies a novel form of time inconsistency and provides a

quantification of its stabilization costs in the United States. This paper extends the framework to address a different question: what are the implications of uncertainty about how markets interpret policy decisions? I show that the resulting precautionary response is itself inefficient: policy responds too weakly overall, and caution can reinforce the uncertainty that motivates it.

The paper thus contributes primarily to the literature initiated by [Brainard \(1967\)](#) on the implications of transmission uncertainty for optimal policy (see, e.g., [Sack \(1998\)](#); [Sargent \(1999\)](#); [Tetlow and von zur Muehlen \(2001\)](#); [Söderström \(2002\)](#); [Giannoni \(2007\)](#) for subsequent contributions). In the standard approach of this literature, the central bank makes its policy decision facing an uncertain but given transmission relationship. Here, due to transmission being partly interpretation-based, the current policy rule itself shapes that relationship. As the analysis below shows, this has distinct implications for optimal policy. In Brainard’s benchmark, uncertainty about the effects of policy makes an attenuated response optimal.¹ In the present model, uncertainty about market interpretation of policy also induces caution under discretion, but this caution is excessive and can generate severe inefficiencies. A related conclusion emerges in [Dupraz et al. \(2023\)](#), where discretionary caution shifts inflation expectations away from the central bank’s target. Here, the inefficiency arises from how systematic policy shapes the interpretation of policy decisions and the uncertainty surrounding transmission.

The analysis also relates to monetary policy under uncertainty about private-sector expectations (see, e.g., [Woodford \(2010\)](#); [Gust et al. \(2025\)](#)). A particularly close contribution is [Caballero and Simsek \(2022\)](#), who show that uncertainty about how markets update their beliefs following a policy decision provides a rationale for gradualism. The argument advanced here qualifies the normative interpretation of this result: while optimal under discretion, this caution is inefficient, since a stronger response improves stabilization without increasing interpretation risk. The present paper also examines a margin absent from their analysis: the composition of the policy rule. By changing what markets infer from decisions and thus the strength of transmission, changes in the policy composition can generate a self-reinforcing feedback between caution and transmission uncertainty.

More broadly, the framework relates to monetary policy with asymmetric beliefs, a connection discussed in [Jimenez-Gimpel and Caravello \(2026\)](#). Particularly relevant is [Stein and Sunderam \(2018\)](#), who show that a privately observed, exogenous interest-rate target and a central bank’s preference for low bond-market volatility generate inefficient gradualism.

¹This “attenuation principle” is not universal. Stronger policy responses can arise under both robust control, as argued early in [Sargent \(1999\)](#), and standard Bayesian uncertainty, as in [Söderström \(2002\)](#). Nevertheless, policymakers often regard caution under transmission uncertainty as a sound practical principle (see, e.g., [Blinder \(1998\)](#)).

Markets anticipate gradual adjustment and respond more strongly to a given rate change, reinforcing policy restraint and potentially supporting multiple equilibria. Both their mechanism and the present one are, at their core, instances of the signal-jamming logic (see, e.g., [Holmstrom \(1982\)](#) and [Stein \(1989\)](#) for early exponents). Here, however, the central bank pursues a standard output-stabilization objective, and caution arises from uncertainty about transmission. A weaker relative response to natural-output assessments can increase that uncertainty, reinforcing the precautionary motive itself.

Roadmap. Section 2 presents the model and the equilibrium at $t \geq 1$ that shapes the market’s inference problem—hence, transmission—at $t = 0$. Section 3 characterizes discretionary policy, the feedback between caution and uncertainty, and the two margins of inefficiency. Appendix A collects proofs of the propositions in the paper while Appendix B contains additional results.

2 Model

This section describes the model, an extension of the framework in [Jimenez-Gimpel and Caravello \(2026\)](#) to accommodate interpretation-based transmission uncertainty. Then, we briefly describe the equilibrium at $t \geq 1$ and its implications for transmission mediated by belief revisions at $t = 0$.

Concretely, let the central bank be uncertain about a structural parameter that shapes the market’s interpretation of policy decisions, the interpretation state. For expositional clarity, the key elements of the basic setup are stated below.

Environment. Consider an economy with fully rigid prices and demand-determined output. Concretely, output is given by a simple IS equation²

$$y_t = \mathbb{E}_t^M y_{t+1} - \gamma(r_t - \rho_t), \quad (1)$$

where $\gamma > 0$, r_t is the interest rate, ρ_t a demand shifter, and $\mathbb{E}_t^M$ the market’s expectation. The central bank minimizes

$$\mathbb{E}^C \sum_{t=0}^{\infty} \beta^t (y_t - y_t^n)^2,$$

²Appendix A in [Jimenez-Gimpel and Caravello \(2026\)](#) derives this economy as a linearization of a standard New Keynesian model with sticky prices and a representative household.

where y_t^n is natural output and $\beta \in (0, 1)$. Between $t = 0$ and $t = 1$, fundamentals follow

$$y_1^n = \varphi_n y_0^n + \varepsilon_n, \quad \rho_1 = \bar{\rho} + \varphi_\rho(\rho_0 - \bar{\rho}) + \varepsilon_\rho,$$

with independent, mean-zero Gaussian innovations $\varepsilon_n, \varepsilon_\rho$, independent also of period-0 variables. Furthermore, $\varphi_n \in (0, 1]$ and $\varphi_\rho \in [0, 1]$ such that $\varphi_n \geq \varphi_\rho$. From date 2 onward, $(y_t^n, \rho_t) = (0, \bar{\rho})$, and this is common knowledge from the outset.

Beliefs. The central bank's information is summarized by a vector of assessments of the economy, $(\hat{y}_0^n, \hat{\rho}_0)$.³ Concretely, the central bank's conditional distribution of the fundamentals is given by

$$\begin{pmatrix} y_0^n \\ \rho_0 \end{pmatrix} \Big| \begin{pmatrix} \hat{y}_0^n \\ \hat{\rho}_0 \end{pmatrix} \lesssim \mathcal{N} \left(\begin{pmatrix} \hat{y}_0^n \\ \hat{\rho}_0 \end{pmatrix}, \begin{pmatrix} \sigma_{\eta,C}^2 & 0 \\ 0 & \sigma_{\varepsilon,C}^2 \end{pmatrix} \right), \quad (2)$$

so the assessments correspond to the central bank's best estimates of the fundamentals. The parameters $\sigma_{\eta,C}^2, \sigma_{\varepsilon,C}^2 > 0$ denote the corresponding (conditional) variances. For later use, define the central bank's assessment errors as $\eta_0 \equiv y_0^n - \hat{y}_0^n$ and $\varepsilon_0 \equiv \rho_0 - \hat{\rho}_0$.

Furthermore, the central bank's ex ante beliefs (i.e., before observing its $t = 0$ assessments) are given by

$$\begin{pmatrix} \hat{y}_0^n \\ \hat{\rho}_0 \end{pmatrix} \lesssim \mathcal{N} \left(\begin{pmatrix} 0 \\ \bar{\rho} \end{pmatrix}, \begin{pmatrix} \sigma_{n,C}^2 & 0 \\ 0 & \sigma_{\rho,C}^2 \end{pmatrix} \right), \quad (3)$$

where $\sigma_{n,C}^2, \sigma_{\rho,C}^2 > 0$. Note that the two assessments are independent under the central bank's ex ante beliefs. The central bank gets no additional information between its policy decisions at $t = 0$ and $t = 1$. This implies

$$\hat{y}_1^n \equiv \mathbb{E}_1^C[y_1^n] = \varphi_n \hat{y}_0^n, \quad \hat{\rho}_1 \equiv \mathbb{E}_1^C[\rho_1] = \bar{\rho} + \varphi_\rho(\hat{\rho}_0 - \bar{\rho}), \quad (4)$$

so the parameters φ_n and φ_ρ also denote the persistence of the central bank's assessments.

The market observes the value of contemporaneous fundamentals (y_t^n, ρ_t) and knows their laws of motion, but it does *not* observe the central bank's estimates. Under the market's $t = 0$ beliefs (i.e., conditional on $t = 0$ fundamentals and s), the estimate vector $(\hat{y}_0^n, \hat{y}_1^n, \hat{\rho}_0, \hat{\rho}_1)$ is jointly Gaussian such that $(\hat{y}_0^n, \hat{y}_1^n) \perp (\hat{\rho}_0, \hat{\rho}_1)$, consistent with the independent nature of $t = 0$ fundamentals and (4). Concretely, market beliefs regarding the central bank's $t = 0$ assessments are such that

$$\begin{pmatrix} \hat{y}_0^n \\ \hat{\rho}_0 \end{pmatrix} \stackrel{M}{\sim} \mathcal{N} \left(\begin{pmatrix} \lambda_n y_0^n \\ \rho_0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & s \end{pmatrix} \right), \quad (5)$$

³A detailed foundation is presented in Appendix B of [Jimenez-Gimpel and Caravello \(2026\)](#).

where $\lambda_n \in [0, 1)$ and $s > 0$. The fundamentals' $t = 1$ innovations $(\varepsilon_n, \varepsilon_\rho)$ are independent of the central bank's $t = 0$ assessments under the market's beliefs. The parameter s introduced in (5) represents the market's *relative* uncertainty about the two estimates,⁴

$$s \equiv \frac{\text{Var}_0^M(\hat{\rho}_0)}{\text{Var}_0^M(\hat{y}_0^n)} \in (0, \infty). \quad (6)$$

From now on, I refer to s as the *interpretation state*. This parameter governs the signal-extraction problem a rate surprise poses: a higher s leads the market to read a given surprise more as news about the central bank's demand assessment and less as news about its natural-output assessment. The market knows (4). In contrast with [Jimenez-Gimpel and Caravello \(2026\)](#), however, the central bank is uncertain about the market's relative uncertainty s and assigns it a distribution F supported on $(0, \infty)$, independent of its assessments and assessment errors.⁵ The consequences of this uncertainty about the interpretation state are the focus of this paper.

Continuation equilibrium and market inference. To characterize market inference at $t = 0$, I first solve for policy and output at subsequent dates, when policy is chosen discretionarily.⁶ The central bank implements its first best $y_t = y_t^n = 0$ at $t \geq 2$. Furthermore, since the environment is linear quadratic, at $t = 1$ it sets $r_1 = \hat{\rho}_1 - \hat{y}_1^n/\gamma$, to achieve expected stabilization given its information, i.e., $y_1 = \hat{y}_1^n + \gamma(\rho_1 - \hat{\rho}_1)$. Consequently, since the central bank's estimates are persistent, those formed at $t = 0$ are informative about output at $t = 1$.

The analysis focuses on linear equilibria, i.e., such that policy has the form

$$r_0 = \bar{\rho} + a(\hat{\rho}_0 - \bar{\rho}) - K\hat{y}_0^n. \quad (7)$$

For $a \neq 0$, write $b := K/a$. Then, we can parameterize a rule by its *scale* a and its *composition* b . Given the linear policy rule, Gaussian updating implies that, for $a \neq 0$, observing a rate

⁴The variances below correspond to the market's prior beliefs (i.e., before observing the policy rate at $t = 0$).

⁵The information structure adapts the subjective-prior construction in Appendix B of [Jimenez-Gimpel and Caravello \(2026\)](#) by making the interpretation state unknown to the central bank. As in that paper's baseline model, a common prior is not imposed (see the discussion there on what the implications of imposing it would be). Uncertainty about s , together with its assumed independence from the central bank's assessments and assessment errors, introduces an additional departure from a common-prior specification. Under the latter, this independence would extend to assessments and fundamentals jointly, so the market's conditional assessment variances could not depend on s . In principle, a common-prior model could accommodate an uncertain s , but that would require links between the interpretation state and the distribution of assessments and errors that we exclude here.

⁶The expressions for policy and output at $t \geq 1$ and the market-inference formula below follow Lemma 1 and Proposition 1, respectively, of [Jimenez-Gimpel and Caravello \(2026\)](#).

r gives

$$y_0 = \varphi_n \lambda_n y_0^n - \gamma(r - \rho_0) - \frac{\chi(b, s)}{a} (r - \mathbb{E}_0^M[r_0]), \quad (8)$$

where $\mathbb{E}_0^M[r_0]$ denotes the market's expectation of the rate before observing its realization and

$$\chi(b, s) := \frac{\varphi_n b + \gamma \varphi_\rho s}{b^2 + s}. \quad (9)$$

The term $\varphi_n \lambda_n y_0^n$ corresponds to the market's expectation of y_1 before observing the rate decision. On the other hand, the last term captures the revision in expected future output following a rate surprise.

If $a = 0$ and $K \neq 0$, the rate fully reveals the natural-output assessment and the belief-revision coefficient is simply φ_n/K . If, on the other hand, the rate is constant ($a = K = 0$), it naturally conveys no information.⁷

The *sensitivity of market beliefs* $\chi(b, s)$ determines the revision in expected future output per unit of policy surprise normalized by the rule's scale. This is independent of a , since scaling the rule changes the policy surprise and the inference coefficient in fully offsetting proportions, leaving the belief revision generated by a given realization of assessments unchanged. Its dependence on b and s determines how strongly a surprise is read as news about each assessment.

Note that since s is unknown to the central bank, generically the sensitivity of market beliefs $\chi(b, s)$ also is. Intuitively, the central bank does not know how the market will interpret its rate decision and thus how much expectations about future output are going to react. Since this reaction shapes current output, this amounts to a form of transmission uncertainty.

3 The Consequences of Transmission Uncertainty

This section studies the policy implications of interpretation-based transmission uncertainty. First, I characterize the discretionary equilibrium and show that uncertainty causes caution, i.e., a less aggressive central bank response to perceived natural output fluctuations than in an environment where s is known and equal to its mean under F . Second, I show that relative to the (linear) commitment benchmark, this discretionary caution can

⁷ Concretely, under those rules, output is given by

$$y_0 = \begin{cases} (\varphi_n + \gamma K) \hat{y}_0^n + \gamma(\rho_0 - \bar{\rho}), & a = 0, K \neq 0, \\ \varphi_n \lambda_n y_0^n + \gamma(\rho_0 - \bar{\rho}), & a = K = 0. \end{cases}$$

be *inefficient* along two distinct margins: the overall scale of the central bank reaction is excessively low and the reaction to natural-output assessments can be too weak as a result of a self-reinforcing feedback between caution and uncertainty. Concretely, I show that the latter inefficiency can produce arbitrarily large losses that can be avoided.

3.1 Discretionary equilibria and cautious policy

A discretionary central bank chooses the current rate after observing its assessments, taking the market’s belief map as given.⁸ Formally, fix the market’s conjectured linear rule and the market belief map it induces. *After observing its assessments*, the central bank solves

$$\arg \min_{r \in \mathbb{R}} \mathbb{E}_0^C \left[(y_0 - y_0^n)^2 \right] \quad (10)$$

subject to (8). Crucially, the function $r \mapsto \mathbb{E}_0^M[y_1 \mid r_0 = r]$ in (8) (or, equivalently, the parameters (a, b)) is taken as given: the central bank does not internalize the fact that in equilibrium the market inference (and thus policy transmission) is shaped by its policy reaction. Loss from $t = 1$ onward is omitted since it is independent of r_0 .

Throughout, I exclude knife-edge equilibria in which the direct and belief-revision effects of a rate adjustment exactly offset across interpretation states, leaving output independent of the rate and the central bank indifferent among all rate choices. This is naturally an empirically irrelevant case.

Before studying the implications of transmission uncertainty, it is useful to recall the form of the equilibrium in the known- s case (Proposition 2 of [Jimenez-Gimpel and Caravello \(2026\)](#)). In that case, there is a *unique* linear discretionary equilibrium with $a_d = 1$ and $b_d(s) \in (0, (1 - \lambda_n \varphi_n)/\gamma)$ such that

$$b_d(s) [\gamma + (1 - \lambda_n) \chi(b_d(s), s)] = 1 - \lambda_n \varphi_n. \quad (11)$$

Intuitively, without transmission uncertainty, certainty equivalence holds: the central bank uses both the direct and belief-revision channels to close its expected output gap. In turn, the market’s sensitivity of beliefs is consistent with the central bank’s policy rule.

The following lemma characterizes the positive discretionary equilibria in the presence of transmission uncertainty.⁹ With uncertainty about s , the marginal effect of a rate adjustment

⁸See [Jimenez-Gimpel and Caravello \(2026\)](#) for a formal definition of a discretionary equilibrium and its game-theoretic counterpart (Appendix C). The adaptation of these to the unknown s case is straightforward.

⁹Unlike in the known- s case, for some parameterizations, negative-composition equilibria may also exist. These are excluded from the main analysis since $b < 0$ implies that the central bank raises the policy rate when its target level of output rises, which is evidently at odds with both the evidence on the direction of

varies across s and is correlated with the stabilization gap in those states. This changes the central bank's optimal response. Concretely, certainty equivalence does not apply.

Lemma 1. *There exists at least one positive discretionary equilibrium. Every discretionary equilibrium has $a_d = 1$, and every positive discretionary equilibrium entails $b_d \in (0, (1 - \lambda_n \varphi_n) \gamma^{-1})$ and satisfies*

$$b_d (\gamma + (1 - \lambda_n) (\mathbb{E}_F[\chi(b_d, s)] + \mathcal{U}_F(b_d))) = 1 - \lambda_n \varphi_n, \quad (12)$$

where

$$\mathcal{U}_F(b) := \frac{\text{Var}_F[\gamma + \chi(b, s)]}{\mathbb{E}_F[\gamma + \chi(b, s)]}.$$

Furthermore, $\mathcal{U}_F(b_d) > 0$ at every positive equilibrium whenever F is nondegenerate.

The lemma above has two components. First, transmission uncertainty does not change the scale a of the discretionary equilibrium. Why is this so? Isolating the demand margin, when $a = 1$ the central bank's conditional expected output gap in interpretation state s is

$$(\gamma + \chi(b, s)) [(\hat{\rho}_0 - \bar{\rho}) - (r_0 - \bar{\rho})].$$

A one-for-one response would thus close the expected gap in *every* interpretation state. This is because the systematic response to the central bank's demand assessment generates no *expected* belief revision under the central bank's beliefs. Thus, on the demand margin the interpretation state is irrelevant for a discretionary central bank.

On the other hand, transmission uncertainty generates a *wedge* that depends on the uncertainty of effective transmission, which is itself shaped by the policy rule. To understand its source, let $h(b, s) \equiv \gamma + \chi(b, s)$ denote total policy transmission. With transmission uncertainty (nondegenerate F), the equilibrium condition can be rewritten as

$$\underbrace{b_d (\gamma + (1 - \lambda_n) \mathbb{E}_F[\chi(b_d, s)]) - (1 - \lambda_n \varphi_n)}_{\text{average stabilization residual}} = - \underbrace{(1 - \lambda_n) b_d \mathcal{U}_F(b_d)}_{\text{state-weighting wedge}} < 0.$$

The LHS is the expected “stabilization residual” on natural output: it would equal zero if the central bank fully stabilized natural output assessments in expectation. However, with transmission uncertainty, states are optimally weighted by $h(b, s)$ on top of their actual probabilities, since a rate adjustment has a larger marginal effect on output when transmission is stronger. At a given b , the stabilization residual per unit of natural-output assessment is larger in precisely those states in which $\chi(b, s)$, and hence $h(b, s)$, is larger.

the effects of monetary policy and the standard prescriptions of monetary theory.

Consequently, high-transmission states (low- s) exert disproportionate influence on the central bank's optimality condition, and thus on the equilibrium. For the central bank to close this transmission-weighted expected gap, its expected stabilization residual must be negative: in other words, the central bank stabilizes *less* than under certainty equivalence. Another way to see this is that the central bank acknowledges that there is a covariance between the gap and the strength of transmission, and it optimally adjusts its policy to account for that.

Furthermore, the factor $1 - \lambda_n$ appears because it determines how strongly the stabilization residual varies with market interpretation. The market observes y_0^n and expects the central bank's assessment to be centered on $\lambda_n y_0^n$, so it anticipates part of the policy response to that assessment. The natural-output component of the policy surprise thus depends on $\hat{y}_0^n - \lambda_n y_0^n$. Since the central bank's assessment is its conditional expectation of natural output,

$$\mathbb{E}^C[\hat{y}_0^n - \lambda_n y_0^n \mid \hat{y}_0^n] = (1 - \lambda_n)\hat{y}_0^n.$$

Thus, holding the policy rule fixed, a higher λ_n means that a higher share of the response to a given natural-output assessment is anticipated by the market (on average from the central bank's perspective). The remaining expected surprise is smaller, making the conditional expected output gap less sensitive to the unknown interpretation state and thus reducing the central bank's exposure to "interpretation risk".

The following Proposition shows that the existence of the wedge above reduces the strength of the central bank reaction to its natural-output assessment in every positive discretionary equilibrium.

Proposition 1 (Uncertainty induces caution). *Let F have finite mean $\bar{s} = \mathbb{E}_F[s]$, and let b_d^F be any positive discretionary equilibrium under F . Let $b_d(\bar{s})$ be the unique positive equilibrium when $s = \bar{s}$ almost surely.*

$$b_d^F \leq b_d(\bar{s}),$$

with strict inequality whenever F is nondegenerate.

Note that since $a_d = 1$ remains true regardless of the extent of uncertainty, a lower composition b necessarily entails a lower reaction coefficient to natural output assessments, hence the term *caution*.

This result is reminiscent of [Brainard \(1967\)](#)'s precautionary principle: uncertainty about the effects of policy can make it optimal to react less aggressively. However, the mechanism behind it is more subtle. In the present environment, the relevant object for transmission is the sensitivity of market beliefs $\chi(b, s)$, which is (a) a nonlinear transformation of the unknown primitive s and (b) an equilibrium object shaped by systematic policy.

The first feature implies that adding uncertainty over s does not merely make the “belief-revision channel” more volatile; it also changes its average strength. Over the equilibrium domain, $\chi(b, s)$ is decreasing and convex in s . Consequently, for a fixed b and a nondegenerate F with mean $\bar{s}$,

$$\mathbb{E}_F[\chi(b, s)] > \chi(b, \bar{s}).$$

Thus, for a fixed b , replacing the degenerate benchmark $s = \bar{s}$ with a nondegenerate distribution raises average transmission. Intuitively, low- s states are those in which a larger fraction of a rate surprise is attributed to the persistent natural-output assessment and is thus especially informative about future output. Moving away from a very low s sharply weakens this inference, while further increases have progressively smaller effects.

In addition, $\mathcal{U}_F(b) > 0$ captures the “state-weighting” channel discussed above. Both forces go in the same direction: they raise the LHS of (12) at any discretionary policy level $b > 0$ relative to the known-transmission benchmark. The central bank faces not only more uncertainty about transmission, but also stronger average transmission and thus equilibrium features a less aggressive response.

Unlike in Brainard (1967), this result does *not* generalize to a comparative static result: higher uncertainty—in the mean-preserving spread sense—does not always entail more caution. This is due to a combination of the non-monotonicity of χ as a function of b and the fact that it is determined in equilibrium. However, when the two fundamentals differ sufficiently in persistence¹⁰, ruling out low- s states does guarantee that higher uncertainty leads to more caution, as shown in Proposition 5 in Appendix B.

The second feature of this novel form of transmission uncertainty, i.e., the endogenous nature of the sensitivity of beliefs and thus of transmission, gives rise to a feedback loop that can generate multiple equilibria. The following example illustrates this possibility.

Example 1 (Multiple Equilibria). *Let $\gamma = 0.5$, $\varphi_n = 1$, $\varphi_\rho = 0$ (i.e., natural output shocks are fully persistent while demand is fully transitory) and $\lambda_n = 0.6$. Further, suppose the central bank entertains a binary distribution over s : $s \in \{10^{-7}, 0.05\}$ with equal probabilities. Then the equilibrium condition (12) has three positive roots, approximately*

$$b_d \in \{0.0054, 0.0488, 0.1435\}.$$

This wide range of equilibrium policy coefficients shows that multiplicity can sustain very different degrees of caution. Since output volatility depends on the equilibrium composition,

¹⁰The persistence relevant for market inference is that of the central bank’s assessments. Since the central bank gets no additional information between the $t = 0$ and $t = 1$ decisions (to avoid introducing separate parameters that do not affect the key mechanism), these persistences coincide with those of the fundamentals.

these equilibria are not generally loss-equivalent. These features highlight the potential for large central-bank losses under transmission uncertainty, as the central bank may end up in a highly inefficient equilibrium according to its *ex ante* criterion. Proposition 3 below formalizes this argument.

To understand the source of equilibrium multiplicity, note that changing b affects not only the *average* strength of the belief revision channel, but also its dispersion across states. A timid reaction can be self-confirming if it leaves the central bank facing a highly uncertain sensitivity of market beliefs $\chi(b, s)$ (and, hence, transmission strength) that in turn validates a low desired b . Conversely, a more aggressive reaction can also be self-confirming if it reduces the dispersion of $\chi(b, s)$ across the relevant states, lowering the “uncertainty wedge” and thus making a higher b optimal. In a nutshell, multiplicity arises because policy aggressiveness and uncertainty about the induced transmission object $\chi(b, s)$ are jointly determined.¹¹

3.2 Two margins of inefficiency: scale and self-reinforcing caution

To assess the efficiency of discretionary policy, a benchmark is needed. A natural candidate is the policy regime such that the central bank chooses a $t = 0$ (linear) policy rule before observing its assessments and simply follows it afterward, i.e., the (linear) commitment regime. In this case, the central bank’s decision internalizes how the rule shapes the market’s belief sensitivity, $\chi(b, s)$. Policy at $t = 1$ remains discretionary, so the comparison isolates the value of committing to the current response rule associated with shaping the market interpretation of policy decisions (as opposed to other potential gains from commitment, such as correcting past mistakes). Formally, the central bank’s loss with commitment is given by¹²

$$\mathcal{L}_C(F) := \inf_{(a, K) \in \mathbb{R}^2} \mathbb{E}^C \left[(y_0 - y_0^n)^2 \right], \quad (13)$$

with market beliefs induced by the announced rule and output determined by (8) for $a \neq 0$ and by the expressions in footnote 7 otherwise.

Since the commitment rule is chosen *ex ante*, the corresponding loss needs to be computed. To do so, note that for $a \neq 0$ and under natural assumptions, Lemma 2 of Jimenez-Gimpel and Caravello (2026) shows that for any given interpretation state s , the central

¹¹In the current stylized model, multiplicity is related to low interpretation states having positive probability, since at high values of s the sensitivity of market expectations becomes very flat in s . Proposition 4 (in Appendix B) provides a formal counterpart to this observation: it gives sufficient conditions for uniqueness of the discretionary equilibrium, one of which requires s to exceed a threshold almost surely. In more general environments, however, the sensitivity of market beliefs may be more strongly state-dependent, and multiplicity may arise even without very low s states.

¹²The infimum is not necessarily attained: nonconstant rules can remain informative as $(a, K) \rightarrow (0, 0)$, while the constant rule conveys no information. The results below do not require the existence of a minimizer.

bank's expected loss is given by

$$L(a, b; s) = (b(\gamma a + \chi(b, s)) - (1 - \lambda_n \varphi_n) - \lambda_n b \chi(b, s))^2 \sigma_{n,C}^2 + (a - 1)^2 \gamma^2 \sigma_{\rho,C}^2 \quad (14)$$

$$+ \underbrace{(\gamma + \chi(b, s))^2 \sigma_{\varepsilon,C}^2 + (1 - \lambda_n \varphi_n + \lambda_n b \chi(b, s))^2 \sigma_{\eta,C}^2}_{\equiv \mathcal{P}(b,s)}.$$

Since by assumption s is independent of the central bank's assessments and their errors, the corresponding ex ante loss in this model is obtained simply by integrating these pointwise losses w.r.t. the distribution of s , i.e., $L_F(a, b) = \int L(a, b; s) dF(s)$.

Note that the first two terms capture systematic stabilization errors—that is, variation in the output gap's conditional mean across realizations of the central bank's assessments. The third and fourth terms, collectively denoted $\mathcal{P}(b, s)$, capture output-gap volatility caused by *assessment* errors.

First, the state-weighting wedge discussed above gives rise to a novel “scale” inefficiency. As shown above, discretion sets the transmission-weighted stabilization gap to zero while leaving the (unweighted) average residual following a unit positive natural-output assessment negative. Holding the composition b_d fixed, increasing the overall aggressiveness of the policy response a raises the stabilization residual by the same amount in every s . This is because, as discussed above, the market adjusts its inference to perfectly offset any scale effect, and thus the latter does not depend on the interpretation state. Thus, a central bank with commitment acknowledges that it can recover some of the lost average stabilization operating solely on the direct rate effect and thus without increasing its exposure to the transmission risk. The following proposition formalizes this argument.

Proposition 2 (Inefficient caution: the risk-free scale margin). *Let b_d be any positive discretionary-equilibrium composition. Holding the composition fixed at b_d , the scale minimizing $L_F(a, b_d)$ is given by*

$$a_F(b_d) = 1 + \left(\frac{b_d^2(1 - \lambda_n)}{\gamma(b_d^2 + \sigma_{\rho,C}^2/\sigma_{n,C}^2)} \right) \mathcal{U}_F(b_d) > 1 \quad (15)$$

whenever F is nondegenerate.

Equation (15) shows that the aforementioned commitment scale correction is proportional to the caution wedge $\mathcal{U}_F(b_d)$ itself, and is attenuated by the same factor $1 - \lambda_n$: the force that makes discretion timid on the composition margin is exactly what commitment offsets in the scale, and both vanish absent transmission uncertainty. This result is important because it shows that, while the positive prediction of the model in a discretionary equilibrium follows [Brainard \(1967\)](#), this caution will be inefficiently high, in the sense that under commitment,

holding composition fixed, the central bank will react more strongly than in a discretionary equilibrium.

The above inefficiency of discretion pertains to the scale a of the policy reaction. The endogeneity of transmission uncertainty creates a distinct and potentially much more severe issue. In a discretionary equilibrium, a smaller composition b can make market interpretation more sensitive to the unknown state, increasing the dispersion of effective transmission and hence the caution wedge that validates the smaller b . This is the same feedback that generates multiplicity in Example 1. Figure 1 illustrates this phenomenon.

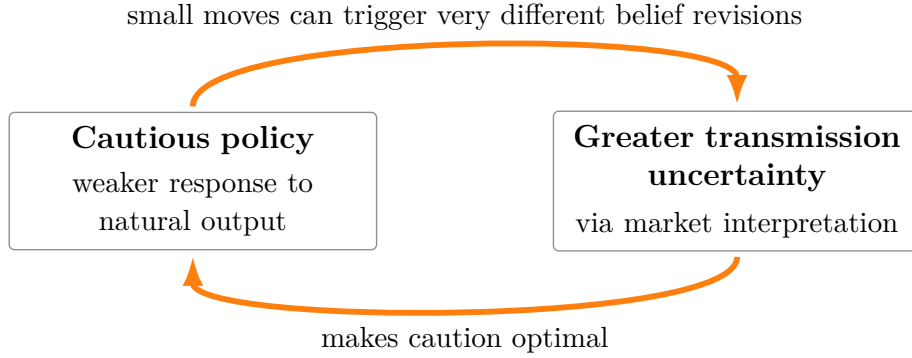


Figure 1: Feedback between policy caution and transmission uncertainty.

The next proposition shows that this feedback can become arbitrarily severe: it can sustain a discretionary equilibrium whose loss diverges even though the primitive variance of s remains bounded, while commitment can guarantee a bounded loss.

Proposition 3 (Self-reinforcing caution can be arbitrarily costly). *Suppose $\varphi_n = 1$. Let $p \in (0, 1)$ satisfy*

$$p > \frac{(1 - \lambda_n)(1 + \varphi_\rho)}{1 + (1 - \lambda_n)(1 + \varphi_\rho)},$$

and consider the binary distribution of s with arbitrary mean $m > 0$ given by

$$F_\ell = p\delta_\ell + (1 - p)\delta_{s_H(\ell)}, \quad s_H(\ell) := \frac{m - p\ell}{1 - p}, \quad 0 < \ell < \frac{m}{p}.$$

For every sufficiently small $\ell > 0$, there exists a positive discretionary equilibrium composition b_ℓ such that, as $\ell \downarrow 0$,

$$b_\ell \longrightarrow 0, \quad \mathcal{U}_{F_\ell}(b_\ell) \longrightarrow \infty.$$

Let $\mathcal{L}_D(F; b)$ denote the central bank's ex ante expected loss at a discretionary equilibrium with composition b , and recall $\mathcal{L}_C(F)$ denotes the value of the commitment problem. Thus,

the associated central bank loss diverges,

$$\mathcal{L}_D(F_\ell; b_\ell) \longrightarrow \infty.$$

The commitment loss instead remains bounded along this family:

$$\limsup_{\ell \downarrow 0} \mathcal{L}_C(F_\ell) < \infty.$$

Consequently, the loss gap between discretion and commitment becomes arbitrarily large:

$$\mathcal{L}_D(F_\ell; b_\ell) - \mathcal{L}_C(F_\ell) \longrightarrow \infty.$$

The result describes a particularly costly discretionary equilibrium. In the low- s state, the market attributes almost the entire surprise to the persistent natural-output assessment. As b_ℓ shrinks, the market correspondingly scales up its inference per unit of policy action, so $\chi(b_\ell, \ell) \sim 1/b_\ell$. This allows an increasingly small response to natural output assessments to remain an equilibrium. But when the rate surprise is instead caused by an error in the central bank's demand assessment, the market also reads it almost fully as persistent natural-output news (since the state s is very low). Since the aggressiveness of the central bank's reaction to its demand assessments is not vanishing, the resulting belief revision, and hence output volatility, become arbitrarily large.¹³

Commitment avoids this outcome because the central bank can announce a fixed rule whose loss remains bounded even as ℓ approaches zero. The diverging loss is thus not an unavoidable consequence of a primitive distribution that becomes arbitrarily volatile: the variance of s remains bounded along the family above (see the proof for details). Instead, it is generated by the self-reinforcing policy response under discretion, which sends b_ℓ toward zero and hence makes market interpretation increasingly sensitive.

Note that the knife-edge persistence $\varphi_n = 1$ is needed for the literal divergence result. Along the constructed sequence, $\ell/b_\ell^2 \rightarrow 0$, and thus $b_\ell \chi(b_\ell, \ell) \rightarrow \varphi_n$. Hence, the belief-revision component of transmission contributes approximately φ_n units of stabilization even as the rate coefficient approaches zero. If $\varphi_n = 1$, inference alone can thus close the conditional expected output gap in the low- s state, allowing an equilibrium with $b_\ell \rightarrow 0$. If $\varphi_n < 1$, this channel leaves $1 - \varphi_n$ that must be covered by the direct policy effect. This places a positive lower bound— $(1 - \varphi_n)/\gamma$ —on the equilibrium response, so the loss cannot

¹³The quadratic loss approximates utility locally around a zero output gap (Appendix A of [Jimenez-Gimpel and Caravello \(2026\)](#)) and thus naturally becomes unreliable for sufficiently large gap fluctuations. However, the potential severity of the self-reinforcing caution mechanism may remain relevant in the nonlinear environment.

diverge for a fixed $\varphi_n < 1$. Hence, the strict divergence requires $\varphi_n = 1$, but the broader economic force does not. Fixing $\varphi_\rho < 1$, a standard continuity argument implies that for each sufficiently small ℓ , taking $\varphi_n < 1$ sufficiently close to one induces a discretionary equilibrium with very large loss. The cost of discretion for $\varphi_n < 1$ according to the central bank's criterion can thus still be very large, even if it does not literally diverge.

The divergence above relies on *future* policy remaining responsive to the central bank's assessments even though its current response to perceived natural output fluctuations becomes increasingly weak. In particular, if in a richer dynamic setting heightened uncertainty is expected to persist and induce caution at future dates as well, weaker future policy responses can reduce the consequences of inferred assessment revisions, potentially offsetting the amplification and its feedback on current caution. However, if an episode of acute uncertainty is instead expected to subside, this offset weakens as current caution can coexist with expectations of more responsive future policy. One can then take the result as a stylized benchmark of such episodes.

Taken together, the results above show that interpretation-based transmission uncertainty has distinct and potentially serious consequences when compared to standard exogenous transmission uncertainty as in [Brainard \(1967\)](#). This is because the policy rule itself shapes the distribution of effective transmission, so the otherwise natural cautionary response to uncertainty can become self-reinforcing and highly inefficient. An ex ante policy rule avoids this outcome, highlighting a novel benefit of commitment in the presence of transmission uncertainty.

4 Conclusion

This paper studies how uncertainty about market interpretation affects monetary policy when transmission is itself shaped by the policy rule. Under discretion, uncertainty attenuates the response to natural-output assessments while leaving the demand response unchanged. The analysis identifies two distinct sources of inefficiency. First, the discretionary policy's overall scale is too low: holding its composition fixed, a proportional increase in both responses can improve stabilization without increasing exposure to interpretation risk. Second, a weak response to natural-output assessments relative to demand assessments can reinforce transmission uncertainty and thus validate itself. This feedback can sustain multiple equilibria and generate arbitrarily large losses, even while the variance of the interpretation state remains bounded. Commitment can avoid these losses.

Thus, the analysis shows that interpretation-based transmission uncertainty has remarkably different implications from those of other standard sources of transmission uncertainty,

i.e., it gives rise to new margins of time inconsistency absent from the known-transmission case characterized in [Jimenez-Gimpel and Caravello \(2026\)](#). A natural question concerns the quantitative importance of these inefficiencies. I plan to tackle this in a future version of the paper.

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A Proofs

Throughout the proofs we write

$$h(b, s) := \gamma + \chi(b, s), \quad \Pi_n(b, s) := 1 - \lambda_n \varphi_n + \lambda_n b \chi(b, s),$$

$$e(a, b, s) := b(\gamma a + \chi(b, s)) - \Pi_n(b, s),$$

so that h and $-\Pi_n$ are the pass-through of demand- and natural-output-assessment errors respectively and e is the coefficient on the natural-output assessment in the output gap. Given these, as shown in Lemma 2 of [Jimenez-Gimpel and Caravello \(2026\)](#), the loss of the central bank for a given realization of the interpretation state s can be written as

$$L(a, b; s) = e(a, b, s)^2 \sigma_{n,C}^2 + (a - 1)^2 \gamma^2 \sigma_{\rho,C}^2 + h(b, s)^2 \sigma_{\varepsilon,C}^2 + \Pi_n(b, s)^2 \sigma_{\eta,C}^2.$$

A.1 Proof of Lemma 1

First, note that the arguments for ruling out $a = 0$ apply to the case with transmission uncertainty. If the conjectured rule also has $K = 0$, the market makes no inference from the rate and the central bank's unique best response loads on both of its assessments, so it cannot coincide with the constant conjecture. If $a = 0$ and $K \neq 0$, the rate fully reveals the natural-output assessment. The total marginal effect of the rate is then $-(\gamma + \varphi_n/K)$. When this term is nonzero, the unique best response has the nonzero demand-assessment coefficient $\gamma/(\gamma + \varphi_n/K)$ and thus cannot coincide with a rule whose demand coefficient is zero. When it is zero, the rate-choice problem is flat so that case is ruled out of the analysis.

Thus, every equilibrium has $a \neq 0$, so its composition $b = K/a$ is well defined. Fix such a conjectured rule (a, b) and define

$$H(a, b, s) := \gamma + \frac{\chi(b, s)}{a}.$$

After observing its assessments, the central bank chooses the current rate while taking the market belief map induced by the conjectured rule as given. Up to terms independent of that rate, its expected loss is

$$\mathbb{E}_F \left[(-H(a, b, s)(r_0 - \bar{\rho}) + h(b, s)(\hat{\rho}_0 - \bar{\rho}) - \Pi_n(b, s)\hat{y}_0^n)^2 \right].$$

As in the known-transmission case, the objective is quadratic in r_0 and its curvature is strictly positive ($\mathbb{E}_F[H(a, b, s)^2] > 0$ by assumption). The first-order condition is necessary

and sufficient and determines the unique global minimizer.

The best response is then simply

$$r_0 - \bar{\rho} = \frac{\mathbb{E}_F[H(a, b, s)h(b, s)]}{\mathbb{E}_F[H(a, b, s)^2]}(\hat{\rho}_0 - \bar{\rho}) - \frac{\mathbb{E}_F[H(a, b, s)\Pi_n(b, s)]}{\mathbb{E}_F[H(a, b, s)^2]}\hat{y}_0^n.$$

Matching its coefficients with the conjectured rule

$$r_0 - \bar{\rho} = a(\hat{\rho}_0 - \bar{\rho}) - ab\hat{y}_0^n$$

gives

$$a = \frac{\mathbb{E}_F[Hh]}{\mathbb{E}_F[H^2]}, \quad ab = \frac{\mathbb{E}_F[H\Pi_n]}{\mathbb{E}_F[H^2]}, \quad (16)$$

where the arguments of H , h , and Π_n are omitted for brevity. Because $h = aH + \gamma(1 - a)$, the first equation in (16) implies

$$\gamma(1 - a) \mathbb{E}_F[H] = 0.$$

Moreover, $\mathbb{E}_F[H]$ cannot be zero. To see this, note that substituting $\chi = a(H - \gamma)$ into Π_n makes the numerator of the RHS of the second equation in (16)

$$\mathbb{E}_F[H\Pi_n] = (1 - \lambda_n\varphi_n) \mathbb{E}_F[H] + \lambda_n ab \mathbb{E}_F[H^2] - \lambda_n \gamma ab \mathbb{E}_F[H].$$

Thus, rearranging that equation yields

$$ab(1 - \lambda_n) \mathbb{E}_F[H^2] = [1 - \lambda_n\varphi_n - \lambda_n\gamma ab] \mathbb{E}_F[H].$$

Since $\lambda_n < 1$ and $a \neq 0$, $\mathbb{E}_F[H] = 0$ implies $b = 0$. At $b = 0$, $\chi(0, s) = \gamma\varphi_\rho$ is constant in s , so H is constant. Consequently, if its mean is zero, it is identically zero and then $\mathbb{E}_F[H^2] = 0$, a contradiction. Hence, $a_d = 1$ at every equilibrium.

At $a_d = 1$, we have $H = h$, and the second equation in (16) becomes

$$b_d = \frac{\mathbb{E}_F[h(b_d, s)\Pi_n(b_d, s)]}{\mathbb{E}_F[h(b_d, s)^2]}, \quad (17)$$

or, equivalently,

$$\mathbb{E}_F[h(b_d, s)e(1, b_d, s)] = 0.$$

To establish existence of a positive solution, define

$$R(b) := b \mathbb{E}_F[h(b, s)^2] - \mathbb{E}_F[h(b, s)\Pi_n(b, s)].$$

Consider first the case of $\varphi_n < 1$. Then, let b_0 be such that $0 < b_0 < (1 - \varphi_n)/\gamma$. Note that $b\chi(b, s) < \varphi_n$ for all $b \in (0, b_0]$. Then,

$$e(1, b, s) = \gamma b + (1 - \lambda_n)b\chi(b, s) - (1 - \lambda_n\varphi_n) \leq \gamma b + (1 - \lambda_n)\varphi_n - (1 - \lambda_n\varphi_n) = \gamma b + \varphi_n - 1 < 0.$$

Since $h > 0$, this gives $R(b) < 0$ for all $b \in (0, b_0]$. On the other hand, if $\varphi_n = 1$, note that

$$e(1, b, s) = \gamma b - \frac{(1 - \lambda_n)(1 - \gamma\varphi_\rho b)s}{b^2 + s}.$$

Thus, $e(1, b, s) > 0$ if and only if

$$\gamma b^3 > s((1 - \lambda_n)(1 - \gamma\varphi_\rho b) - \gamma b). \quad (18)$$

Because $1 - \lambda_n > 0$, for all sufficiently small $b > 0$,

$$(1 - \lambda_n)(1 - \gamma\varphi_\rho b) - \gamma b \geq \frac{1 - \lambda_n}{2},$$

and hence (18) implies

$$s < \underbrace{\left(\frac{2\gamma}{1 - \lambda_n}\right)}_{\equiv C} b^3.$$

Thus, for all sufficiently small $b > 0$, $e(1, b, s) > 0$ requires $s \in (0, Cb^3)$.

Furthermore, since $h > 0$,

$$\begin{aligned} [he]_+ &\leq h\gamma b \\ &= \gamma b(\gamma + \chi(b, s)) \\ &= \gamma^2 b + \gamma b\chi(b, s) \\ &\leq \gamma^2 b + \gamma. \end{aligned}$$

Thus, for sufficiently small b there is some finite M independent of s such that $[he]_+ \leq M$.

Hence, the above implies

$$\mathbb{E}_F[(he)_+] \leq M \times \mathbb{P}(s \in (0, Cb^3)).$$

Since the support of F is contained in $(0, +\infty)$ by assumption, it has no probability mass at zero. Hence, $\mathbb{P}(s \in (0, Cb^3)) \rightarrow 0$ as $b \downarrow 0$.

Now let's work with the negative part of he . Let $\delta > 0$ be such that

$$p_\delta := F([\delta, \infty)) > 0.$$

Note that

$$\chi(b, s) - \gamma\varphi_\rho = \frac{b(1 - \gamma\varphi_\rho b)}{b^2 + s},$$

and thus

$$\sup_{s \geq \delta} |\chi(b, s) - \gamma\varphi_\rho| \leq \frac{b|1 - \gamma\varphi_\rho b|}{\delta} \rightarrow 0$$

as $b \downarrow 0$. Hence, $h(b, s) \rightarrow \gamma(1 + \varphi_\rho)$ as $b \downarrow 0$ uniformly over $s \geq \delta$. Similarly,

$$\sup_{s \geq \delta} \left| \frac{s(1 - \gamma\varphi_\rho b)}{b^2 + s} - 1 \right| \rightarrow 0,$$

so as $b \downarrow 0$ $e(1, b, s) \rightarrow -(1 - \lambda_n)$ uniformly over $s \geq \delta$. Consequently, as $b \downarrow 0$,

$$h(b, s)e(1, b, s) \rightarrow -\gamma(1 + \varphi_\rho)(1 - \lambda_n) < 0$$

uniformly over $s \geq \delta$. Thus, there exist $\xi > 0$ and sufficiently small $b_1 > 0$ such that

$$h(b, s)e(1, b, s) \leq -\xi$$

whenever $0 < b < b_1$ and $s \geq \delta$. It follows that

$$\mathbb{E}_F [he\mathbf{1}_{\{s \geq \delta\}}] \leq -\xi p_\delta.$$

Thus, for sufficiently small b ,

$$R(b) \leq M \times \mathbb{P}(s \in (0, Cb^3)) - \xi p_\delta.$$

Since the first term converges to zero while the second is fixed, this shows that $R(b) < 0$ for sufficiently small $b > 0$ also in the case of $\varphi_n = 1$.

Now, at $\bar{b} := (1 - \lambda_n \varphi_n) \gamma^{-1} > 0$,

$$\bar{b}h(\bar{b}, s) - \Pi_n(\bar{b}, s) = (1 - \lambda_n)\bar{b}\chi(\bar{b}, s).$$

Hence,

$$R(\bar{b}) = (1 - \lambda_n)\bar{b} \mathbb{E}_F [h(\bar{b}, s)\chi(\bar{b}, s)] > 0$$

since the expression inside the expectation is positive pointwise. The function R is clearly continuous on $(0, \infty)$. Since $R(b) < 0$ for all sufficiently small $b > 0$ and $R(\bar{b}) > 0$, the intermediate-value theorem gives a root $b_d \in (0, \bar{b})$.

It remains to establish the stated properties of every positive solution. For $b > 0$, $\chi(b, s) > 0$ and thus $h(b, s) > \gamma > 0$. Write $\bar{h}_F(b) := \mathbb{E}_F[h(b, s)]$ and note that¹⁴ $\text{Var}[h(b, s)] = \text{Var}[\chi(b, s)]$ and

$$\mathbb{E}_F[h\chi] = \bar{h}_F(b) \mathbb{E}_F[\chi(b, s)] + \text{Var}[\chi(b, s)].$$

Using these together with the identity $\mathbb{E}_F[h^2] = \bar{h}_F(b)^2 + \text{Var}[\chi(b, s)]$ and the definition of Π_n , the fixed point (17) becomes

$$b_d \left(\bar{h}_F(b_d)^2 + \text{Var}[\chi(b_d, s)] \right) = (1 - \lambda_n \varphi_n) \bar{h}_F(b_d) + \lambda_n b_d \left(\bar{h}_F(b_d) \mathbb{E}_F[\chi(b_d, s)] + \text{Var}[\chi(b_d, s)] \right).$$

Dividing by $\bar{h}_F(b_d) > 0$ and collecting the terms involving $\mathcal{U}_F(b_d) = \text{Var}[\chi(b_d, s)]/\bar{h}_F(b_d)$ gives

$$b_d \left(\bar{h}_F(b_d) + (1 - \lambda_n) \mathcal{U}_F(b_d) \right) = 1 - \lambda_n \varphi_n + \lambda_n b_d \mathbb{E}_F[\chi(b_d, s)].$$

Substituting $\bar{h}_F(b_d) = \gamma + \mathbb{E}_F[\chi(b_d, s)]$ and rearranging yields the equivalent form

$$b_d (\gamma + (1 - \lambda_n) (\mathbb{E}_F[\chi(b_d, s)] + \mathcal{U}_F(b_d))) = 1 - \lambda_n \varphi_n,$$

which is exactly (12). Since $\lambda_n < 1$, $\mathbb{E}_F[\chi(b_d, s)] > 0$, and $\mathcal{U}_F(b_d) \geq 0$, the coefficient multiplying b_d is strictly greater than γ . Hence,

$$b_d < \frac{1 - \lambda_n \varphi_n}{\gamma}.$$

Finally, note that the above implies

$$h_s(b, s) = \frac{b(\gamma \varphi_\rho b - \varphi_n)}{(b^2 + s)^2} < 0$$

since $b < (1 - \lambda_n \varphi_n) \gamma^{-1}$ and $\varphi_n \geq \varphi_\rho$. Thus $h(b_d, s)$ is strictly decreasing in s . A non-degenerate F consequently induces a nondegenerate effective-transmission coefficient, so $\text{Var}[h(b_d, s)] > 0$. Since $\bar{h}_F(b_d) > 0$, it follows that $\mathcal{U}_F(b_d) > 0$ as stated.

¹⁴All variances are with respect to the distribution F .

A.2 Proof of Proposition 1

Note first that if F is degenerate, the two economies coincide and thus the conclusion holds with equality. Consequently, from here onward assume that F is nondegenerate. Let $\bar{s} := \mathbb{E}_F[s]$, and let b_d^F be any positive equilibrium under F .

Because $h(b, s) > 0$, the Radon-Nikodym theorem allows us to define the tilted probability measure ν_b by

$$\frac{d\nu_b}{d\mu_F} = \frac{h(b, s)}{\mathbb{E}_F[h(b, s)]},$$

where μ_F is the probability measure induced by F . Then

$$0 = \mathbb{E}_F[h(b_d^F, s)e(1, b_d^F, s)] = \mathbb{E}_F[h(b_d^F, s)] \mathbb{E}_{\nu_{b_d^F}}[e(1, b_d^F, s)],$$

so

$$(1 - \lambda_n)b_d^F \mathbb{E}_{\nu_{b_d^F}}[\chi(b_d^F, s)] = 1 - \lambda_n\varphi_n - \gamma b_d^F. \quad (19)$$

Let $b_d(\bar{s})$ denote a positive equilibrium in the economy in which the central bank thinks $s = \bar{s}$ almost surely. By Proposition 2 of [Jimenez-Gimpel and Caravello \(2026\)](#), it is unique and satisfies

$$(1 - \lambda_n)b_d(\bar{s})\chi(b_d(\bar{s}), \bar{s}) = 1 - \varphi_n\lambda_n - \gamma b_d(\bar{s}).$$

Moreover, every positive equilibrium has $b \in (0, (1 - \lambda_n\varphi_n)\gamma^{-1})$ as proved above. Hence, since $\varphi_n \geq \varphi_\rho$,

$$\chi_s(b, s) = \frac{b(\gamma\varphi_\rho b - \varphi_n)}{(b^2 + s)^2} < 0, \quad \chi_{ss}(b, s) = \frac{2b(\varphi_n - \gamma\varphi_\rho b)}{(b^2 + s)^3} > 0,$$

so $\chi(b, s)$ is decreasing and convex in s , and the same is true of $h(b, s)$. Hence, Jensen's inequality implies

$$\mathbb{E}_{\nu_b}[\chi(b, s)] \geq \chi(b, \mathbb{E}_{\nu_b}[s]),$$

with strict inequality unless F is degenerate. Moreover, $h(b, s)$ is decreasing in s , so $\mathbb{E}_{\nu_b}[s] < \mathbb{E}_F[s] = \bar{s}$, and since $\chi(b, s)$ is decreasing in s ,

$$\chi(b, \mathbb{E}_{\nu_b}[s]) > \chi(b, \bar{s}).$$

Applying these inequalities at $b = b_d^F$ in (19) yields

$$\gamma b_d^F + (1 - \lambda_n)b_d^F \chi(b_d^F, \bar{s}) < 1 - \lambda_n\varphi_n.$$

Note that $b \mapsto \gamma b + (1 - \lambda_n)b\chi(b, \bar{s})$ is increasing on the relevant domain (see the proof of Proposition 2 of [Jimenez-Gimpel and Caravello \(2026\)](#)) and equals $1 - \lambda_n\varphi_n$ at $b_d(\bar{s})$. Thus, $b_d^F \leq b_d(\bar{s})$, strictly unless F is degenerate.

A.3 Proof of Proposition 2

We first state the commitment objective used in the proof. The interpretation state s is independent of the central bank's assessments and their estimation errors. Conditional on each s , therefore, the same substitution and orthogonality argument used in the proof of Lemma 2 of [Jimenez-Gimpel and Caravello \(2026\)](#) applies. Integrating the resulting conditional loss with respect to F gives, for $a \neq 0$,

$$L_F(a, b) = \mathbb{E}_F \left[(b(\gamma a + \chi(b, s)) - (1 - \lambda_n\varphi_n) - \lambda_n b\chi(b, s))^2 \sigma_{n,C}^2 + (a - 1)^2 \gamma^2 \sigma_{\rho,C}^2 \right. \\ \left. + (\gamma + \chi(b, s))^2 \sigma_{\varepsilon,C}^2 + (1 - \lambda_n\varphi_n + \lambda_n b\chi(b, s))^2 \sigma_{\eta,C}^2 \right]. \quad (20)$$

This is the transmission-uncertainty counterpart of (14) needed here.

Fix $b = b_d$ for the rest of the proof. For an arbitrary scale a , the first term of (20) can then be written as

$$\sigma_{n,C}^2 \mathbb{E}_F \left[(\gamma b_d(a - 1) + e(1, b_d, s))^2 \right].$$

The two assessment-error terms are independent of a for fixed b . Thus, the part of the objective that depends on a is

$$\sigma_{n,C}^2 \mathbb{E}_F \left[(\gamma b_d(a - 1) + e(1, b_d, s))^2 \right] + \gamma^2 \sigma_{\rho,C}^2 (a - 1)^2.$$

This is a strictly convex quadratic in a . Its first-order condition is

$$\gamma^2 \left(\sigma_{\rho,C}^2 + b_d^2 \sigma_{n,C}^2 \right) (a - 1) = -\gamma b_d \sigma_{n,C}^2 \mathbb{E}_F [e(1, b_d, s)].$$

It remains to evaluate the mean stabilization error on the right-hand side. The discretionary equilibrium condition is $\mathbb{E}_F[h(b_d, s)e(1, b_d, s)] = 0$, so, letting $\bar{h}_d := \mathbb{E}_F[h(b_d, s)] > 0$,

$$\mathbb{E}_F[e(1, b_d, s)] = -\frac{\text{Cov}_F(h(b_d, s), e(1, b_d, s))}{\bar{h}_d}.$$

Both $h(b_d, s)$ and $e(1, b_d, s)$ depend on s only through $\chi(b_d, s)$, with coefficients 1 and $(1 - \lambda_n)b_d$ respectively, so

$$\text{Cov}_F(h(b_d, s), e(1, b_d, s)) = (1 - \lambda_n)b_d \text{Var}[\chi(b_d, s)] = (1 - \lambda_n)b_d \text{Var}[h(b_d, s)],$$

and hence $\mathbb{E}_F[e(1, b_d, s)] = -(1 - \lambda_n)b_d\mathcal{U}_F(b_d)$. Substituting into the first-order condition gives the unique optimal scale *fixing* $b = b_d$:

$$a_F(b_d) - 1 = \frac{(1 - \lambda_n)\mathcal{U}_F(b_d)b_d^2}{\gamma(b_d^2 + \sigma_{\rho, C}^2/\sigma_{n, C}^2)},$$

which is exactly (15). The denominator is clearly positive and $\mathcal{U}_F(b_d) > 0$ whenever F is nondegenerate (Lemma 1). Hence, $a_F(b_d) > 1$ whenever F is nondegenerate.

A.4 Proof of Proposition 3

We first establish the bad discretionary outcome. Throughout this proof, $\Pi_n(b, s) = 1 - \lambda_n + \lambda_n b\chi(b, s)$, since $\varphi_n = 1$.

Note that under the beliefs in the proposition, $s_H(\ell) \rightarrow m/(1-p) > 0$ as $\ell \downarrow 0$. Moreover,

$$\text{Var}(s) = p(\ell - m)^2 + (1-p)(s_H(\ell) - m)^2 = \frac{p}{1-p}(m - \ell)^2 \longrightarrow \frac{p}{1-p}m^2.$$

Thus, the variance of the interpretation state remains bounded along this family. Define the discretionary equilibrium residual

$$\Psi_\ell(b) := \mathbb{E}_{F_\ell}[h(b, s)e(1, b, s)]. \quad (21)$$

The positive discretionary equilibria are precisely the positive zeros of Ψ_ℓ .

Those zeros are found by studying the residual at coefficients of order $\ell^{1/3}$. To see why this is the relevant candidate scale, if b^2 is large relative to ℓ , the low-state contribution to $\Psi_\ell(b)$ contains the term $-(1 - \lambda_n)\ell/b^3$, while its remaining leading terms and the high-state contribution are of order one. Balancing these terms suggests that b^3 should be of order ℓ . Accordingly, set $t := \ell^{1/3}$ and evaluate the equilibrium residual at $b = ct$, where $c > 0$ is for now arbitrary; coefficients of this form are not being presumed to be equilibria. In the low state $s = \ell = t^3$, factoring t^2 out of the denominator gives

$$h(ct, t^3) = \gamma + \frac{ct + \gamma\varphi_\rho t^3}{c^2 t^2 + t^3} = \gamma + \frac{1}{t} \frac{c + \gamma\varphi_\rho t^2}{c^2 + t}.$$

For fixed $c > 0$,

$$\frac{1}{c^2 + t} = \frac{1}{c^2} - \frac{t}{c^4} + O(t^2),$$

so

$$\begin{aligned} h(ct, t^3) &= \gamma + \frac{1}{t} \left(\frac{1}{c} - \frac{t}{c^3} + O(t^2) \right) \\ &= \frac{1}{t} \left[\frac{1}{c} + t \left(\gamma - \frac{1}{c^3} \right) + O(t^2) \right]. \end{aligned}$$

Multiplying this expansion by ct and rearranging terms gives

$$\begin{aligned} \Pi_n(ct, t^3) &= 1 - \lambda_n + \lambda_n ct \chi(ct, t^3) \\ &= 1 - \lambda_n + \lambda_n ct (h(ct, t^3) - \gamma) \\ &= 1 - \lambda_n + \lambda_n \left(1 + t \left(\gamma c - \frac{1}{c^2} \right) + O(t^2) - \gamma ct \right) \\ &= 1 - \frac{\lambda_n t}{c^2} + O(t^2). \end{aligned}$$

Thus, the stabilization residual becomes

$$e(1, ct, t^3) = ct h(ct, t^3) - \Pi_n(ct, t^3) = t \left(\gamma c - \frac{1 - \lambda_n}{c^2} + O(t) \right)$$

and then

$$h(ct, t^3) e(1, ct, t^3) = \gamma - \frac{1 - \lambda_n}{c^3} + O(t). \quad (22)$$

In the high state, let $s_H^* := m/(1 - p) > 0$. Since $\ell = t^3$,

$$s_H(t^3) = s_H^* - \frac{p}{1 - p} t^3 = s_H^* + O(t^3).$$

It follows that

$$\begin{aligned} h(ct, s_H(t^3)) &= \gamma + \frac{ct + \gamma \varphi_\rho s_H(t^3)}{c^2 t^2 + s_H(t^3)} \\ &= \gamma(1 + \varphi_\rho) + O(t). \end{aligned}$$

Thus, high-state transmission remains bounded, and since $ct \chi(ct, s_H(t^3)) = O(t)$ we have $\Pi_n(ct, s_H(t^3)) = 1 - \lambda_n + O(t)$, so

$$e(1, ct, s_H(t^3)) = -(1 - \lambda_n) + O(t).$$

Multiplying the last two expansions gives

$$h(ct, s_H(\ell)) e(1, ct, s_H(\ell)) = -(1 - \lambda_n) \gamma(1 + \varphi_\rho) + O(t). \quad (23)$$

These expansions are uniform when c ranges over a compact subset of $(0, \infty)$. Indeed, on such a set both c and $1/c$ are bounded, while $s_H(t^3)$ is bounded away from zero for all sufficiently small t . Consequently, both the low-state denominator $c^2 + t$ and the high-state denominator $c^2 t^2 + s_H(t^3)$ are bounded away from zero on the chosen compact set. Thus, the constants in the bounds on the remainder terms can be chosen uniformly over that set (independently of c). It follows that

$$\Psi_\ell(c\ell^{1/3}) \longrightarrow \Phi(c) := \gamma[p - (1-p)(1-\lambda_n)(1+\varphi_\rho)] - \frac{p(1-\lambda_n)}{c^3}. \quad (24)$$

Since $\lambda_n < 1$, Φ is strictly increasing, and because $p > (1-\lambda_n)(1+\varphi_\rho)/[1 + (1-\lambda_n)(1+\varphi_\rho)]$ is equivalent to $p - (1-p)(1-\lambda_n)(1+\varphi_\rho) > 0$, it has a unique positive zero c_* such that

$$c_*^3 = \frac{p(1-\lambda_n)}{\gamma(p - (1-p)(1-\lambda_n)(1+\varphi_\rho))}.$$

Fix $\epsilon \in (0, c_*)$. Because Φ is strictly increasing and $\Phi(c_*) = 0$, uniform convergence in (24) implies that, for all sufficiently small ℓ ,

$$\Psi_\ell((c_* - \epsilon)\ell^{1/3}) < 0, \quad \Psi_\ell((c_* + \epsilon)\ell^{1/3}) > 0.$$

Continuity therefore gives a positive zero between these two points. Since ϵ is arbitrary, we may select such a zero b_ℓ satisfying

$$b_\ell = c_*\ell^{1/3} + o(\ell^{1/3}) \longrightarrow 0. \quad (25)$$

We next calculate effective transmission at this equilibrium. Since $b_\ell^2/\ell \rightarrow \infty$, the low-state transmission coefficient satisfies

$$h(b_\ell, \ell) \sim \frac{1}{b_\ell},$$

while $h(b_\ell, s_H(\ell)) = O(1)$. Consequently,

$$\bar{h}_{F_\ell}(b_\ell) := \mathbb{E}_{F_\ell}[h(b_\ell, s)] \sim \frac{p}{b_\ell}, \quad (26)$$

$$\mathbb{E}_{F_\ell}[h(b_\ell, s)^2] \sim \frac{p}{b_\ell^2}. \quad (27)$$

Subtracting the square of the mean from the second moment gives

$$\text{Var}[h(b_\ell, s)] \sim \frac{p(1-p)}{b_\ell^2}.$$

Therefore

$$\mathcal{U}_{F_\ell}(b_\ell) = \frac{\text{Var}[h(b_\ell, s)]}{\bar{h}_{F_\ell}(b_\ell)} \sim \frac{1-p}{b_\ell} \longrightarrow \infty. \quad (28)$$

We now translate these asymptotics into statements about the central bank's ex ante loss. Since at any positive discretionary equilibrium $a_d = 1$, the corresponding loss is

$$\mathcal{L}_D(F; b) := \mathbb{E}_F[e(1, b, s)^2] \sigma_{n,C}^2 + \mathbb{E}_F[h(b, s)^2] \sigma_{\varepsilon,C}^2 + \mathbb{E}_F[\Pi_n(b, s)^2] \sigma_{\eta,C}^2.$$

Note that $e(1, b, \cdot)$ only depends on s via $\chi(b, \cdot)$, and it is affine in it with slope $(1 - \lambda_n)b$, so

$$\text{Var}[e(1, b, s)] = (1 - \lambda_n)^2 b^2 \text{Var}[h(b, s)], \quad \text{Cov}(h(b, s), e(1, b, s)) = (1 - \lambda_n)b \text{Var}[h(b, s)].$$

Thus, the equilibrium condition $\mathbb{E}_F[h(b, s)e(1, b, s)] = 0$ can be written as $\mathbb{E}_F[e(1, b, s)] = -(1 - \lambda_n)b\mathcal{U}_F(b)$, and hence

$$\mathbb{E}_F[e(1, b, s)^2] = (1 - \lambda_n)^2 b^2 \left(\text{Var}[h(b, s)] + \mathcal{U}_F(b)^2 \right).$$

At $b = b_\ell$ the first term of the loss is therefore bounded, converging to $(1 - \lambda_n)^2(1 - p)\sigma_{n,C}^2$ by (26)–(28). The third is bounded as well: by Lemma 1, $b_\ell < \gamma^{-1}$, so $\gamma\varphi_\rho b_\ell \leq \varphi_\rho \leq \varphi_n$ and hence $b_\ell\chi(b_\ell, s) \leq \varphi_n = 1$, giving $\Pi_n(b_\ell, s) \in [1 - \lambda_n, 1]$ for every ℓ . However, by (27),

$$\mathbb{E}_{F_\ell}[h(b_\ell, s)^2] \sim \frac{p}{b_\ell^2} \longrightarrow \infty.$$

Hence $\mathcal{L}_D(F_\ell; b_\ell) \rightarrow \infty$ whenever $\sigma_{\varepsilon,C}^2 > 0$.

Finally, fix any positive rule $(a, K) = (1, b)$ whose coefficient does not vary with ℓ . At the low support point, $h(b, \ell) \rightarrow \gamma + 1/b$, while at the high support point it converges to a finite value ($e(1, b, s)$ and $\Pi_n(b, s)$ are likewise bounded uniformly in s at the fixed rule). Hence, the ex ante expected loss generated by this fixed rule is bounded as $\ell \downarrow 0$. Since the rule is feasible under commitment and $\mathcal{L}_C(F_\ell)$ is the infimum over all feasible linear rules, it follows that

$$\limsup_{\ell \downarrow 0} \mathcal{L}_C(F_\ell) < \infty.$$

Together with $\mathcal{L}_D(F_\ell; b_\ell) \rightarrow \infty$, this proves that the discretion–commitment loss gap diverges. Because $\mathcal{L}_C(F_\ell)$ is defined as an infimum, the argument does not require the commitment

problem to attain its minimum.

B Additional Results

Proposition 4 (Sufficient Conditions for Uniqueness of Discretionary Equilibrium). *Suppose*

$$\varphi_n > 2\varphi_\rho(1 - \lambda_n\varphi_n) \quad (29)$$

and

$$s > \max \left\{ \frac{\varphi_n^2}{4\gamma^2(1 + \varphi_\rho)}, \frac{(1 - \lambda_n\varphi_n)^2}{\gamma^2} \frac{3\varphi_n - 2\varphi_\rho(1 - \lambda_n\varphi_n)}{\varphi_n - 2\varphi_\rho(1 - \lambda_n\varphi_n)} \right\} \quad (30)$$

F-a.s. Then the linear discretionary equilibrium is unique.

Proof. By Lemma 1, every discretionary equilibrium is such that $a_d = 1$ and its composition b_d satisfies

$$\mathbb{E}_F[h(b_d, s)e(1, b_d, s)] = 0.$$

We first rule out negative compositions. Fix $b < 0$. Note that

$$h(b, s) = \gamma + \chi(b, s) = \frac{\gamma b^2 + \varphi_n b + \gamma(1 + \varphi_\rho)s}{b^2 + s},$$

and the numerator is an upward-opening parabola in b and thus bounded below by

$$\gamma(1 + \varphi_\rho)s - \frac{\varphi_n^2}{4\gamma}.$$

Thus, (30) implies $h(b, s) > 0$ *F*-a.s. Hence, since $b < 0$,

$$e(1, b, s) = \gamma b + (1 - \lambda_n)b\chi(b, s) - (1 - \lambda_n\varphi_n) \leq \lambda_n\gamma b - (1 - \lambda_n\varphi_n) < 0.$$

The strict support restriction therefore implies $h(b, s) > 0$, *F*-almost surely. Since $e(1, b, s) < 0$ for $b < 0$, their product has strictly negative expectation. Hence, no negative composition can satisfy the equilibrium condition.

Thus, it only remains to show the uniqueness of the positive equilibrium. Let

$$\bar{b} := \frac{1 - \lambda_n\varphi_n}{\gamma}.$$

Lemma 1 shows that every positive equilibrium belongs to $(0, \bar{b})$. For simplicity, define

$$A(b) := \mathbb{E}_F[h(b, s)], \quad B(b) := \mathbb{E}_F[h(b, s)^2].$$

Since

$$e(1, b, s) = b \left((1 - \lambda_n) h(b, s) + \lambda_n \gamma \right) - (1 - \lambda_n \varphi_n),$$

the equilibrium condition can be written as

$$b \left(\lambda_n \gamma + (1 - \lambda_n) \frac{B(b)}{A(b)} \right) = 1 - \lambda_n \varphi_n. \quad (31)$$

Fix $b \in (0, \bar{b})$. The fact that $b < \bar{b}$ and (29) imply

$$\varphi_n - 2\gamma\varphi_\rho b \geq \varphi_n - 2\varphi_\rho(1 - \lambda_n \varphi_n) > 0.$$

Further,

$$\begin{aligned} h_s(b, s) &= -\frac{b(\varphi_n - \gamma\varphi_\rho b)}{(b^2 + s)^2} < 0, \\ h_b(b, s) &= \frac{(\varphi_n - 2\gamma\varphi_\rho b)s - \varphi_n b^2}{(b^2 + s)^2}, \\ h_{bs}(b, s) &= \frac{b^2(3\varphi_n - 2\gamma\varphi_\rho b) - s(\varphi_n - 2\gamma\varphi_\rho b)}{(b^2 + s)^3}. \end{aligned}$$

Note that $h_{bs}(b, s) < 0$ if and only if

$$s > b^2 \left(\frac{3\varphi_n - 2\gamma\varphi_\rho b}{\varphi_n - 2\gamma\varphi_\rho b} \right). \quad (32)$$

Differentiating and rearranging, it can be shown that the RHS of (32) is strictly increasing in b on $(0, \bar{b})$, and (30) implies it holds at $\bar{b}$. Thus, $h_{bs}(b, s) < 0$ for all $b \in (0, \bar{b})$. Further, note that $h_b > 0$ if and only if

$$s > b^2 \left(\frac{\varphi_n}{\varphi_n - 2\gamma\varphi_\rho b} \right),$$

which is implied by (32) because $\varphi_n > \gamma\varphi_\rho b$ in the relevant domain since $\varphi_n \geq \varphi_\rho$. Hence, $h_b(b, s) > 0$ for all $b \in (0, \bar{b})$. The fact that both $h(b, s)$ and $h_b(b, s)$ are decreasing in s implies

$$\text{Cov}_F(h(b, s), h_b(b, s)) \geq 0. \quad (33)$$

Moreover, on the relevant domain,

$$h(b, s) = \gamma(1 + \varphi_\rho) + \frac{b(\varphi_n - \gamma\varphi_\rho b)}{b^2 + s} > \gamma(1 + \varphi_\rho).$$

Further, use $b/(b^2 + s) \leq 1/(2\sqrt{s})$ and the first component of (30) to obtain

$$\begin{aligned} \frac{b(\varphi_n - \gamma\varphi_\rho b)}{b^2 + s} &\leq \frac{\varphi_n b}{b^2 + s} \\ &\leq \frac{\varphi_n}{2\sqrt{s}} \\ &< \gamma\sqrt{1 + \varphi_\rho} \\ &\leq \gamma(1 + \varphi_\rho). \end{aligned}$$

Thus,

$$\gamma(1 + \varphi_\rho) < h(b, s) < 2\gamma(1 + \varphi_\rho)$$

F -a.s. Consequently,

$$B(b) < 2\gamma(1 + \varphi_\rho)A(b) < 2A(b)^2.$$

As per the argument above, $A'(b) = \mathbb{E}_F[h_b(b, s)] > 0$. Using that plus $B'(b) = 2\mathbb{E}_F[h(b, s)h_b(b, s)]$ and (33), we get

$$B'(b)A(b) - B(b)A'(b) = A'(b) \left(2A(b)^2 - B(b) \right) + 2A(b) \text{Cov}_F(h(b, s), h_b(b, s)) > 0.$$

Hence, $B(b)/A(b)$ is strictly increasing on $(0, \bar{b})$. The LHS of (31) is thus strictly increasing in b in the relevant domain and thus there is at most one positive equilibrium. Since we already established that there are no negative equilibria under the given conditions and clearly $b = 0$ is not an equilibrium, the equilibrium is unique. □

Proposition 5 (General mean-preserving spreads and caution). *Let F and G be distributions over s with a finite common mean such that F is a mean-preserving spread of G . Suppose (29) holds and (30) holds F -a.s. and G -a.s. Then, the unique linear discretionary equilibrium under F has a weakly lower composition than that under G ,*

$$b_d^F \leq b_d^G.$$

Furthermore, whenever the spread is nontrivial, the inequality is strict.

Proof. By Proposition 4, the equilibria under F and G are unique and satisfy

$$0 < b < \frac{1 - \lambda_n \varphi_n}{\gamma},$$

so we consider only b in this domain. For simplicity, write

$$c := \gamma(1 + \varphi_\rho), \quad \alpha := b(\varphi_n - \gamma\varphi_\rho b) > 0,$$

so that

$$h(b, s) = c + \frac{\alpha}{b^2 + s}.$$

For $H \in \{F, G\}$, let

$$y_H := \mathbb{E}_H[(b^2 + s)^{-1}], \quad z_H := \mathbb{E}_H[(b^2 + s)^{-2}].$$

Then

$$\frac{\mathbb{E}_H[h(b, s)^2]}{\mathbb{E}_H[h(b, s)]} = \frac{c^2 + 2c\alpha y_H + \alpha^2 z_H}{c + \alpha y_H}. \quad (34)$$

Since $(b^2 + s)^{-1}$ and $(b^2 + s)^{-2}$ are strictly convex in s , the fact that F is a mean-preserving spread of G implies

$$y_F \geq y_G, \quad z_F \geq z_G, \quad (35)$$

with strict inequalities whenever the spread is nontrivial.

The RHS of (34), as a function of (y_H, z_H) , has derivatives

$$\frac{\alpha(c^2 - \alpha^2 z_H)}{(c + \alpha y_H)^2} \quad \text{and} \quad \frac{\alpha^2}{c + \alpha y_H}, \quad (36)$$

respectively. The second is positive. To sign the first, note that by the argument in the proof of Proposition 4,

$$0 < \frac{\alpha}{b^2 + s} < c.$$

Thus, $\alpha^2 z_H < c^2$ and the first derivative in (36) is positive as well. Thus, by (34) and (35), for every b in the relevant domain,

$$\frac{\mathbb{E}_F[h(b, s)^2]}{\mathbb{E}_F[h(b, s)]} \geq \frac{\mathbb{E}_G[h(b, s)^2]}{\mathbb{E}_G[h(b, s)]}, \quad (37)$$

and strictly whenever the spread is nontrivial.

As mentioned in the proof of Proposition 4, the equilibrium under either distribution $H \in \{F, G\}$ is the unique solution to

$$b \left(\lambda_n \gamma + (1 - \lambda_n) \frac{\mathbb{E}_H[h(b, s)^2]}{\mathbb{E}_H[h(b, s)]} \right) = 1 - \lambda_n \varphi_n.$$

The LHS is strictly increasing under both distributions by the proof of Proposition 4, while (37) shows it is strictly increasing in the distribution (in the MPS sense). Thus, $b_d^F \leq b_d^G$ and the inequality is strict when the spread is nontrivial. $\square$